DETERMINATION OF THE RADIATIVE CONTRIBUTION TO THE EFFECTIVE THERMAL CONDUCTIVITY OF A GRANULAR MEDIUM UNDER VACUUM CONDITIONS

E.S. Hütter, N.I. Kömle
Space Research Institute of the Austrian Academy of Sciences, Graz, Austria

Abstract

The effective thermal conductivity of a granular material can be given as the total of the conductivities related to the heat transfer mechanisms which are taking place. These are conduction via the participating phases, convection possibly occurring in the fluid part and radiation. Under normal conditions (atmospheric pressure and room temperature) by and large the fluid part is controlling the effective thermal conductivity and the influence of radiation only becomes significant at elevated temperatures. This could change for the case of a granular material under vacuum conditions where no fluid phase is present. This is a setting of basic interest to the field of planetology, since many airless planetary bodies, like the Moon are covered with granular material. In literature no information on experimentally determined values of the radiant conductivity under vacuum conditions and at room temperature is available. Therefore we report on thermal conductivity measurements performed on granular material under vacuum conditions and at room temperature in order to try to determine the magnitude of the influence of radiative heat transfer in such a system. As granular material two sorts of glass spheres were used. Both are made of the same material and have the same grain size. The only difference between the two samples lies in their ability to absorb/emit radiation. The effective thermal conductivity of these materials was determined in a pressure range from $3 \times 10^{-4}$ mbar up to approximately atmospheric pressure. Additionally the radiative conductivity of the system was estimated using two different models. Our experimental results indicate that in granular material under vacuum conditions radiation heat transfer cannot be neglected, even at room temperature.

Nomenclature

$\psi$ Porosity

$d_p$ Particle diameter

$\sigma_k$ Thermal conductivity measurement error, Wm$^{-1}$K$^{-1}$

$l$ Length

$P_{Heat}$ Heating power, Wm$^{-1}$

1 Introduction

The surface layers of many planetary bodies of our solar system are occupied by granular silicates, the so-called "regolith". Since these layers are forming the boundary region between the bodies' interior and their outer environment (space or atmosphere), the thermophysical properties of those materials, especially the thermal conductivity, are key parameters in the field of planetology.

The effective thermal conductivity granular material has been of interest since the beginning of the last century e.g. Kannulik and Martin 1933 [4]. Extensive investigations of this property
also under reduced air pressures have been carried out e.g. by Watson 1964 [15], Glaser and Wechsler 1965 [3], Merril 1969 [8] and Presley and Christensen 1997 [11][10]. Watson and Merril also investigated the radiative influence. Theoretical work regarding radiative heat transfer in beds of packed spheres have been done e.g. by Schotte 1960 [13] or Mohamad et al. 1994 [9]. Numerical modeling of heat transfer in porous media and radiative properties like e.g. absorption coefficients has been done to great extend by numerous authors in the last decades [12], [5]. However most of the work is dedicated to systems at elevated temperatures, where it is known that radiation is a dominant process since it is strongly temperature dependent. The research work done for this article is in particular dedicated to the regolith-covered airless solar system bodies like the Moon, where loosely packed granular material of varying size is exposed to a vacuum environment [14]. In such an environment heat is transported via conduction and radiation. The investigation of heat transfer in such a system and of the possible magnitudes of the individual transfer modes is vital for planetary research and for designing thermal sensors which are scheduled for future lunar missions [7]. Therefore thermal conductivity experiments have been performed on granular material under differing pressure conditions in an attempt to determine the magnitude of radiative heat transfer in granular materials under space conditions. The used samples, outlined in Section 3, were glass beads of the same size and material which are only differing in their radiative properties. The experimentally determined radiant conductivity values were compared to values estimated from two different models. These two models and the related theory are described in Section 2. Information regarding the measurement method and the measurement setup is also given in Section 3. The respective results are presented in Section 4 and discussed in Section 5.

2 Theory

2.1 Heat Transfer in Granular Material

In the case of a two phase granular system at room temperature consisting of solid particles and air an effective thermal conductivity composed of several contributions can be given [12], [5]. The system is considered to be dry and no phase changes are occurring. In this case the overall effective thermal conductivity can be expressed as the sum of the contributions from the conductivities of the single phases, radiation and also convection possibly occurring in the gas:

\[ k_{\text{eff}} = (k_{\text{solid}} + k_{\text{gas}}) + k_{\text{rad}} + k_{\text{con}} \]  

(1)

Under normal pressure conditions the gaseous part is controlling the effective thermal conductivity. Conduction via the matrix of the solid particles and radiation are very small in comparison to the gaseous contribution. This is due to the fact that, depending on the contact resistance, the contact points between the particles act as thermal capacitors. This reduces the conductive contribution from the solid phase [11]. Furthermore, the radiative conductivity, which is proportional to \( T^3 \), is negligible for particles with a size of 1 mm at temperatures below 600 K [13]. However, if the gaseous phase is removed (vacuum) the effective thermal conductivity of a granular medium decreases drastically, because of the disappearance of the gas related contributions. The expression for the effective thermal conductivity is in this case reduced to

\[ k_{\text{eff}} = k_{\text{solid}} + k_{\text{rad}} \]  

(2)

That is, only the contributions from the solid phase and radiation are left, in which radiation can be the most efficient transfer mechanism. Both contributions are depending on particle properties and particle size. The solid conductivity depends on the material, the size and
the surface texture. The radiative part is influenced by the size, the emissivity and most significantly by the temperature.

2.2 The Radiative Contribution

**Simple Model:** First one of the simplest models for estimating the radiative conductivity was considered. This model includes radiative heat transfer between two parallel plates of different temperature \((T \text{ and } (T + dT))\). The plates are separated by a distance \(l\). The heat flow per unit area taking place between the two plates can be expressed as

\[
\dot{q}_{\text{rad}} = \varepsilon \sigma \left[ (T + dT)^4 - T^4 \right]
\]

This equation can be expanded after \(T\) to give

\[
\dot{q}_{\text{rad}} = 4\varepsilon \sigma T^3 dT
\]

Furthermore it is also possible to express the radiative heat transfer given in Equation (4) in terms of thermal conductivity:

\[
\dot{q}_{\text{rad}} = k_{\text{rad}} \frac{dT}{dz} \approx k_{\text{rad}} \frac{dT}{l}
\]

Equating relation (4) and relation (5) leads to

\[
k_{\text{rad}} \frac{dT}{l} = 4\varepsilon \sigma T^3 dT
\]

From this a radiative conductivity can be found:

\[
k_{\text{rad}} = 4\varepsilon \sigma T^3 l
\]

Relation (7) indicates that the temperature has a very strong influence on the radiative conductivity. Furthermore the radiative conductivity depends on the emissivity of the plates and the separation distance between the plates. Now, for estimating the radiant conductivity, it was considered that the separation distance \(l\) is approximately corresponding to the grain size of the used glass beads.

**Model of Schotte:** Secondly, a model for the radiative conductivity in a bed of packed spheres, which was developed by Schotte [13], was used. In this model the influences of particle size, porosity, emissivity, particle thermal conductivity and temperature are taken into account.

\[
k_{\text{rad}} = \frac{1 - \psi}{(1/k_{\text{solid}}) + (1/k_r)} + \psi k_r
\]

\[
k_r = 0.229 \psi \varepsilon d_p T^3
\]

**Experimental Determination:** Finally the radiative influence was determined from thermal conductivity measurement data. The used samples of glass spheres are only differing in their ability to absorb/emit radiation (Paragraph 3). Therefore the radiative conductivity should be obtainable from effective thermal conductivity measurement data by using Relation (2). For equal particle conductivity the difference in radiative conductivity can be determined as

\[
\Delta k_{\text{rad}} = k_{\text{eff},2} - k_{\text{eff},1} = k_{\text{rad},2} - k_{\text{rad},1}
\]
Table 1: Properties and important values regarding the used glass spheres.

<table>
<thead>
<tr>
<th>$d_p$ mm</th>
<th>principal constituents</th>
<th>$\varepsilon$</th>
<th>$\rho$ kg/l</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00 - 1.25</td>
<td>70% SiO$_2$, 13.1% Na$_2$O, 8.7% CaO, 4.1% MgO</td>
<td>0.95</td>
<td>2.5</td>
<td>0.38</td>
</tr>
</tbody>
</table>

3 Experiments

Material  In this work two kinds of glass beads of the same grain size and the same material were used. The only difference between the two sorts of glass beads is that one sample is nearly transparent and the other one is pigmented black. So these two samples only differ in their ability to absorb/emit radiation. In a system of glass spheres in the mm size range radiative transfer should mainly take place between the particles [15]. Both sorts are shown in Figure 1. Furthermore important properties regarding the samples are listed in Table 1. There is to mention that the used porosity value corresponds to a bed of randomly packed spheres. Furthermore, since there was no actual emissivity or transmissive data available for these materials, the radiant conductivity was estimated by using an average emissivity value for glass and applying the simple model and the model of Schotte. In the case of the measurements in vacuum, where heat transfer only occurs via contact points and radiation, differences in the effective thermal conductivity should only be due to differences in their respective emissivities.

Measurement Method  For the measurements a commercial thermal conductivity probe was used. This measurement instrument, which uses non steady state heat transfer to determine the thermal conductivity, has already been used very effectively to measure the thermal conductivity of granular materials, like sand, soil and glass spheres [16], [1], [6]. The advantage of this technique is its swiftness and the relatively easy and straightforward way of deriving the thermal conductivity from the measurement data. This measurement method is based on the solution of the problem of an infinitely long line heat source embedded in an infinitely extended medium. It was first stated by Carslaw and Jaeger [2]. Therefore an infinitely long and thin line in an infinitely expanding medium is considered. The embedded line liberates a constant amount of heat at a constant rate. For this geometry a solution for the temperature distribution depending on time and space can be found from the equation of heat conduction. For sufficiently long heating times it is possible to perform a series expansion on this solution. Now, if the

Figure 1: Glass spheres used as for the thermal conductivity measurements. On the right side the pigmented sample is shown. On the left side the nearly transparent sample can be seen.
Temperature gradient is examined after sufficiently long heating the following simple relation can be found:

\[ \Delta T = \frac{P_{\text{Heat}}}{4\pi k} \ln \frac{t_1}{t_2} \]  

(11)

This equation gives a linear relation between the temperature rise caused by constant heating of a line source and the natural logarithm of the heating time. The slope of this curve is depending on the used heating power \( P_{\text{Heat}} \) and the thermal conductivity of the surrounding medium. Therefore, if the temperature rise is measured with time and the heating power is known, the thermal conductivity can be easily calculated by linear regression. The actually used thermal conductivity probe is shown at the bottom of Figure 2.

**Measurement Setup** The thermal conductivity measurements were performed in a vacuum chamber which is capable of producing a vacuum of approximately \( 10^{-5} \text{ mbar} \). Furthermore it is also possible to establish different pressure levels by means of an adjustable nitrogen gas flow. The measurement environment and the measurement setup are shown in Figure 2 along with the used commercial thermal conductivity probe produced by the company Hukseflux. For all measurements same heating power and interval were used. This also accounts to assure that every measurement was taken under as alike conditions as possible. The measurements were performed under approximately room temperature (297K). The maximum temperature gradients induced by the measurement procedure constituted about 6K. The measurement procedure for each of the two samples was the same. The material was put in a container and placed in the vacuum chamber which was subsequently closed and evacuated. After reaching a pressure of \( 3 \times 10^{-4} \text{ mbar} \) and thermal equilibrium in the sample a measurement was performed. The pressure in the chamber was noted and monitored during the measurement. During a measurement all relevant measurement data were stored in a data logger. After completing

Figure 2: Setup for the performed measurements. Top left: In the center of the picture the used vacuum chamber can be seen. The pumps are located underneath the chamber. On the right side the nitrogen gas cylinder can be spotted. The yellow rotary-control serves the adjustment of the nitrogen gas flow. Top Right: Glass sphere sample with the conductivity probe mounted and placed in the vacuum chamber. Bottom: Usual commercial thermal conductivity probe [1].
the measurement the thermal conductivity was calculated by a logger-related software using the approximation outlined in Paragraph 3. Then measurement data were transferred from the data logger to the computer where a closer inspection was done. For each pressure level three measurements were performed. After that the pressure was increased by increasing the nitrogen gas flow manually. Then the procedure was repeated for the next pressure level. On the whole measurements were performed at twelve pressure levels.

4 Results

The determined thermal conductivity values as function of the common logarithm of the chamber pressure are shown in Figure 4. The different samples are indicated by different colors: blue for the dark glass spheres, red for the nearly transparent glass spheres. Both measurement curves show the typical, well known s-shape arising from the decrease of the gas pressure which results in a decrease of the thermal conductivity. It can be observed that, for pressures above \( 10^9 \) mbar, the effective thermal conductivity is almost the same. At lower pressures a clear split up between the effective thermal conductivity values of the two samples can be noticed. This difference at lower pressures is associated with differences in the radiative conductivity. For the four lowest pressure levels this difference as a function of the common logarithm of the pressure is shown in Figure 4. The conductivity value determined at approximately atmospheric pressure was 0.169 Wm\(^{-1}\)K\(^{-1}\) for the "clear" glass spheres and 0.172 Wm\(^{-1}\)K\(^{-1}\) for the black glass spheres. For the lowest pressure level of \( 3 \times 10^{-4} \) mbar the derived effective thermal conductivity was 0.0154 Wm\(^{-1}\)K\(^{-1}\) for the transparent spheres and 0.0226 Wm\(^{-1}\)K\(^{-1}\) for the pigmented spheres. Each conductivity value given in this section was calculated as the arithmetic mean of three single measurement values. The respective relative measurement errors were of the order \( 10^{-3} \). The overall temperature during the tests was approximately 297K. In Table 3 the mean conductivity difference obtained from the conductivity values at the three lowest pressure levels is summarized. Out of this a mean difference of 0.0076 Wm\(^{-1}\)K\(^{-1}\) was derived. This value is approximately half of the obtained effective thermal conductivity value of the nearly transparent glass spheres. If a "worst case" scenario is considered, which means that the measurement errors are contributing in a way, that the difference reaches its minimum, still a value of 0.0056 Wm\(^{-1}\)K\(^{-1}\) is obtained. On the other hand, in the case of a "best case" scenario, the difference in the thermal conductivity is 0.0096 Wm\(^{-1}\)K\(^{-1}\). By virtue of the theory of heat transfer in granular material which was outlined in Section 2, we interpret this difference to be a consequence of a difference in the radiative conductivities of the two samples compared. Moreover also an estimation of the radiative conductivity of the glass spheres using the models outlined in Paragraph 2.2 is given. The respective material properties needed for the estimate are given in Table 1. In Table 2 the measured thermal conductivity difference is compared to the radiative conductivity estimated from the models. It is striking that the values derived from the simple model are relatively close to the measured ones. Strangely enough the radiative conductivity determined from Schotette's model, considering the radiative influence in a bed of packed spheres is significantly smaller than the measured difference.

Table 2: Measured conductivity difference and radiative thermal conductivity values estimated from two models.

<table>
<thead>
<tr>
<th>Type</th>
<th>( k_{rad} )</th>
<th>( \sigma_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured</td>
<td>0.0076</td>
<td>±0.002 (measurement uncertainty)</td>
</tr>
<tr>
<td>estimated by simple model</td>
<td>0.0061</td>
<td>±0.0007 (due to grain size distribution)</td>
</tr>
<tr>
<td>estimated by model of Schotte</td>
<td>0.0026</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3: Effective thermal conductivity values as a function of the common logarithm of the pressure.

Figure 4: Differences in the effective thermal conductivity as a function of the common logarithm of the pressure taking into account the respective measurement errors.

Table 3: Determination of the mean difference in the thermal conductivity between the two glass sphere samples. Errors of the thermal conductivity values are all of the order 0.001 Wm$^{-1}$K$^{-1}$.

<table>
<thead>
<tr>
<th>$p$, mbar</th>
<th>$k_{\text{eff, clear}}$</th>
<th>$k_{\text{eff, black}}$</th>
<th>$\Delta k_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^{-4}$</td>
<td>0.015</td>
<td>0.023</td>
<td>0.008</td>
</tr>
<tr>
<td>$8 \times 10^{-4}$</td>
<td>0.016</td>
<td>0.024</td>
<td>0.008</td>
</tr>
<tr>
<td>$8 \times 10^{-3}$</td>
<td>0.016</td>
<td>0.023</td>
<td>0.007</td>
</tr>
<tr>
<td><strong>Mean:</strong></td>
<td><strong>0.0076 ± 0.002</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 Discussion

Recapitulatory, an attempt has been made to determine the magnitude of radiative heat transfer due to changed optical properties in a granular material under vacuum conditions where this is one of two heat transfer mechanisms at hand. Additionally also estimations of the radiative conductivity of the respective samples using two different models have been done. The determined values are valid for wavelengths resembling the temperatures in the range of 300±3K (for a black body obeying Planck’s law this accords to wavelengths around 9.6 µm).

Our experimental results suggest that on the investigated size scales radiative heat transfer cannot be neglected even at room temperature. The magnitude of the found difference is of the order of the measured overall effective thermal conductivity. According to our assumptions the bed of blackened spheres has a higher effective thermal conductivity due to improved radiative heat transfer than the bed of "clear" glass spheres. The values for the radiative thermal conductivity estimated from the two different models are smaller. In the case of the model developed by Schotte, which reflects the real situation best, the estimate is approximately three times smaller than the experimentally determined radiative conductivity difference. An advancement and better characterisation of the results will be possible when the spectral behaviour and the precise emissivities of the glass spheres are determined experimentally, since the results are very sensitive to these properties. But nevertheless the value determined from the model of Schotte is just one magnitude smaller than the measured effective thermal conductivity. Thus it does
not really change the fact that radiative heat transfer should not be neglected when studying the heat transfer in a granular system under vacuum conditions.

References


